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Effect of magnetic field on electron neutrino sphere in pulsars

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Abstract

We study the neutrino interaction rates through charged as well as neutral current weak interactions in hot, dense magnetized matter. At densities near the neutrino sphere, matter in the presence of intense magnetic field is polarized and electrons and protons typically occupy the lowest Landau level. The weak interaction rates in such a situation are severally modified leading to a sizable change in neutrino opacity and consequently on the location of the neutrino sphere which also gets distorted. This has implications for pulsar velocity explanations.

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Recently there has been a lot of interest [1] - [5] in the study of neutrino transport in dense, hot nuclear matter in the presence of strong magnetic fields reported to be present in young pulsars in connection with a possible mechanism to explain the recoil velocities of pulsars. Specifically, an asymmetry arising due to parity violation in a strong magnetic field can lead to asymmetry in the explosion and recoil of newly formed neutron stars. The asymmetry could for example, arise in the standard weak interactions of neutrinos [1] - [3], due to neutrino magnetic moment [6] or due to matter induced neutrino oscillations in the presence of magnetic field [4] - [5]. In order to explain the large velocities of neutron stars one needs an asymmetry of roughly one percent in the radiated neutrinos which could arise due to a number of neutrino scattering reactions on polarised electrons and hadron of the magnetized medium. A particularly attractive mechanism to explain pulsar kicks is suggested by Kusenko and Segre [4] and makes use of resonant MSW conversion.The way this mechanism works is as follows:

The high densities and temperatures reached in proto-neutron star stage results in the trapping of neutrinos within their neutrino-spheres i.e. the surface from which the optical depth of neutrinos become one. The location of this neutrino-sphere is flavor dependent because of the fact that electron neutrinos interact both through charged as well as neutral currents, whereas the tau and mu neutrinos have only the neutral current interactions. This results in the tau neutrino escaping from deeper layers of star (due to their large mean free path) where the temperature is high. The electron neutrinos trapped between the tau and electron neutrino sphere can then oscillate through resonant conversion to tau neutrinos and become able to

escape-thereby making the tau neutrino sphere to effectively coincide with the resonance surface. The presence of magnetic field modifies the potentials relevant for describing neutrino conversion and leads to the distortion of resonance surface for MSW conversion through polarisation of the magnetised medium. This leads to tau neutrino emission in different directions from regions of different temperatures, therefore with different energies and thus leading to pulsar kicks. For this mechanism to work magnetic fields of strength $B \geq 10^{14}G$ are required and such high magnetic fields in the context of supernova collapse have indeed been proposed in the literature [7].

Such strong magnetic fields would affect the neutrino opacities by modifying the electron neutrino absorption and scattering cross-sections on nucleons and leptons which determine the electron neutrino mean free path. The location of the ν_e sphere would thus get shifted and distorted due to asymmetry in the cross-section. This effect on the dominant absorption reaction $\nu_e + n \rightarrow p + e^-$ has been calculated in the literature [8] by modifying the phase space distribution of the final state electrons only and by leaving the matrix elements unchanged. In the presence of magnetic fields of the order of $10^{16}G$, the motion of the degenerate electrons at densities likely to be present near the neutrino sphere ($Y_e\rho \sim 10^{10} - 10^{11}g/cc$)is in-fact quantised and the electrons typically occupy the lowest Landau energy level leading to total polarisation of electrons opposite to the direction of the magnetic field. Because of charge neutrality viz $\bar{n}_e = n_p$, the protons too are forced to occupy the lowest Landau level. In this situation , the matrix elements for the absorption and scattering processes from nucleons and leptons get modified and have to be calculated by using the exact wave functions for

electrons and protons by solving the Dirac equation in the magnetic field namely for the lowest Landau level $\nu = 0$. Further in order to make numerical estimates of the neutrino mean free path in hot magnetised matter, we require to know the composition of nuclear matter. This is done by considering electrically neutral hot n-p-e matter in β- equilibrium in the presence of magnetic field. The nucleons here are treated non-relativistically and at the densities of interest, strong interaction effects can be safely ignored.

The processes that contribute to ν_e opacity are the absorption

$$\nu(p_1) + n(p_2) \rightarrow p(p_3) + e^-(p_4) \tag{1}$$

and the scattering

$$\begin{aligned} \nu(p_1) + n(p_2) &\rightarrow n(p_3) + \nu(p_4) & (2)\\ \nu(p_1) + p(p_2) &\rightarrow p(p_3) + \nu(p_4) & (3)\\ \nu(p_1) + e^-(p_2) &\rightarrow e^-(p_3) + \nu(p_4) & (4) \end{aligned}$$

processes.

The cross-section per unit volume of matter or the inverse mean free path is given by

$$\frac{\sigma(E_1)}{V} = \lambda^{-1}(E_1) = \frac{1}{2E_1}\Pi_{n=2,3,4} d\rho_n W_{fi} f_2(E_2)(1 - f_3(E_3))(1 - f_4(E_4)) \tag{5}$$

where $d\rho_n = \frac{d^3p_n}{(2\pi)^3 2E_n}$ is the density of states of particles with four momenta $p_n, f_n(E_n)$ are the particle distribution functions which in thermal equilibrium are given by the usual Fermi-Dirac distributions $f_n(E_n) = 1/(1 + exp(\frac{E_n - \mu_n}{T}))$ and $(1 - f_n(E_n))$ accounts for the Pauli Blocking factor for the final state particles. The transition rate W_{fi} is

$$W_{fi} = (2\pi)^4 \delta^4(P_f - P_i) \left|M\right|^2 \tag{6}$$

where $|M|^2$ is the squared matrix element summed over initial and final spins. In the presence of magnetic field, the density of states are replaced by

$$d\rho_n = \sum_{\nu}(2-\delta_{\nu,0})\int_{-\infty}^{\infty}\int_{-\frac{eBL_X}{2}}^{\frac{eBL_X}{2}}\frac{dp_{nz}dp_{ny}}{(2\pi)^2 2E_n} \tag{7}$$

where the sum over the Landau levels is to be performed. If the magnetic field is not strong enough to force particles in the lowest $\nu = 0$ Landau state, the matrix element remains essentially unchanged [9] and one has to only take the proper phase space into consideration. In the presence of quantising magnetic field, a situation likely to be present for strong fields at densities and temperatures under consideration, the electrons occupy the lowest Landau level and the matrix elements have to be calculated by using the exact wave functions of relativistic electrons and protons in a magnetic field. In a gauge in which the vector potential $A = (0, xB, 0)$, the quantum states are specified by the quantum numbers $p_y, p_z, \nu ands$ and the energy is given by [10]

$$E = \sqrt{m^2 + p_z^2 + 2\nu eB} \tag{8}$$

and the positive energy electron spinor in $\nu = 0$ state is given by

$$u_e = \begin{pmatrix} 0 \\ E_e + m_e \\ 0 \\ -p_{eZ} \end{pmatrix} \tag{9}$$

electrons being alligned antiparallel to the direction of the magnetic field which is along the Z axis. The x-component of the momentum is not conserved now and the transition rate becomes

$$W_{fi} = (2\pi)^3\delta(E_f - E_i)\delta(P_{fz} - P_{iz})\delta(P_{fy} - P_{iy})\,|M|^2 \tag{10}$$

$V = L_x L_y L_Z$ is the normalization volume. Squared matrix element summed over initial and final spins for processes (1)-(4) in the Standard model is given by

$$|M|^2 = 32G_F^2\Big[(C_V+C_A)^2 p_1.p_2 p_3.p_4+(C_V-C_A)^2 p_2.p_4 p_1.p_3-m_2 m_3(C_V^2-C_A^2)p_1.p_4\Big] \tag{11}$$

the vector and the axial vector couplings are given by

$Process$	C_V	C_A
$\nu n \rightarrow pe$	$g_V = 1$	$g_A \approx 1.22$
$\nu n \rightarrow n\nu$	$-g_V/2$	$-g_A/2$
$\nu p \rightarrow p\nu$	$1/2 - 2sin^2\theta_W \approx 0$	$g_A/2$
$\nu e \rightarrow e\nu$	$1/2 + 2sin^2\theta_W$	$1/2$

During the Kelvin-Helmholtz cooling phase of the proto-neutron star, the density is much less than the nuclear density. The nucleons are non-relativistic and non-degenerate, the electrons remain relativistic. The neutrinos are non-degenerate and the elastic approximation in the scattering processes is quite reliable. The phase space integrals in eqn(5) can then be carried out analytically [11] and we obtain the mean free paths as

$$\lambda_{abs}^{-1}(0) = \frac{G_F^2}{\pi}(g_V^2 + 3g_A^2)E_\nu^2 n_N \frac{1}{1+exp(E_\nu + Q - \mu_e)\beta} \tag{12}$$

$$\lambda_n^{-1}(0) = \frac{G_F^2}{4\pi}(g_V^2 + 3g_A^2)E_\nu^2 n_N \tag{13}$$

$$\lambda_p^{-1}(0) = \frac{3G_F^2}{4\pi} g_A^2 E_\nu^2 n_p \tag{14}$$

$$\lambda_e^{-1}(0) = \frac{2G_F^2}{3\pi^3}(C_V^2 + C_A^2)\mu_e^2 T E_\nu^2 \tag{15}$$

Considering now the magnetic field that affects only the electrons and which is not strong enough to confine the electrons to the lowest Landau state, the matrix elements for the processes remain essentially unchanged and modifying only the phase space, the important absorption cross-section turns out to be [8]

$$\begin{aligned}\frac{\sigma_{abs}(B)}{V} &\sim \frac{G_F^2}{2\pi}(g_V^2+3g_A^2)eB\frac{1}{1+exp(E_\nu+Q-\mu_e)\beta} \\ &\quad (E_\nu+Q)\sum(2-\delta_{\nu,0})\frac{1}{\sqrt{(E_\nu+Q)^2-m_e^2-2\nu eB}} \qquad (16)\end{aligned}$$

The cross-section for electron neutrino scattering in hot dense matter in the presence of magnetic field has been derived in [2]. The other processes remain unaffected by the magnetic field.

In the interesting case of strongly quantised magnetic field when both electrons and protons in stellar matter are polarised with their spins aligned anti parallel and parallel to the direction of the magnetic field respectively, the matrix elements for the above processes are calculated using exact wave functions for $\nu = 0$ Landau state .For non-relativistic nucleons and relativistic leptons, we get for the matrix element squared

$$\begin{aligned}M_{abs}^2 &= 8G_F^2 m_2 m_3(E_4+p_{4z})\Big[(g_V+g_A)^2+4g_A^2E_1+(g_V-g_A)^2+4g_A^2E_1Cos\theta\Big] \\ &\quad exp\Big[-\frac{1}{2eB}(p_{1x}+p_{2x})^2+(p_{3y}+p_{4y})^2\Big] \qquad (17) \\ M_{p\nu}^2 &= 16G_F^2{m_2}^2(p_1.p_4+2p_{1Z}.p_{4Z}) \\ &\quad exp\Big[-\frac{1}{2eB}(p_{4x}-p_{1x})^2+(p_{4y}-p_{1y})^2\Big] \qquad (18) \\ M_{e\nu}^2 &= 16G_F^2\Big[(C_V^2+C_A^2)\Big[(E_1E_4+p_{1Z}p_{2Z})(E_2E_3+p_{2Z}p_{4Z})\end{aligned}$$

$$
\begin{aligned}
&-(E_1 p_{4Z} + E_4 p_{1Z})(E_2 p_{3Z} + E_3 p_{2Z})\Big] \\
&+2C_V C_A\Big[(E_1 E_4 + p_{1Z} p_{4Z})(E_2 p_{3Z} + E_3 p_{2Z}) \\
&-(E_1 p_{4Z} + E_4 p_{1Z})(E_2 E_3 + p_{2Z} p_{3Z})\Big]\Big] \\
&exp\Big[-\frac{1}{2eB}(p_{4x} - p_{1x})^2 + (p_{4y} - p_{1y})^2\Big] \qquad (19)
\end{aligned}
$$

The scattering cross-sections are now given by

$$
\begin{aligned}
\frac{\sigma_{scatt}(B)}{V} &= \lambda_{scatt}^{-1} \\
&= \frac{1}{2E_1 L_X}\int\frac{d^3p_4}{(2\pi)^3 2E_4}\int\frac{dp_{2Y}dp_{2Z}}{(2\pi)^2 2E_2}\int\frac{dp_{3Y}dp_{3Z}}{(2\pi)^2 2E_3} \\
&\quad (2\pi)^3\delta(P_Y)\delta(P_Z)\delta(E)|M|^2 \\
&\quad f_2(E_2)(1 - f_3(E_3))(1 - f_4(E_4)) \qquad (20)
\end{aligned}
$$

and the absorption cross-section is obtained by exchanging the fourth and the second particles. The integrals over y-component of the momenta can be done by using the y-component momentum conserving delta function. The rest of the integrals can be done by using $\delta(E)$ and $\delta(P_Z)$ in the limit of non-degenerate, non-relativistic nucleons. The exponential functions in the squared matrix elements can be approximated by one in this limit and further in the limit of elastic scattering approximation. Carrying out the integrals we obtain

$$
\begin{aligned}
\frac{\sigma_{abs}(B)}{V} &= \lambda_{abs}^{-1}(B) \sim \frac{G_F^2}{4\pi} eBn_N\Big[\{(g_V + g_A)^2 + 4g_A^2\} + \{(g_V + g_A)^2 - 4g_A^2\}cos\theta\Big] \\
&\quad \frac{1}{1 + exp\{-(E_\nu + Q - \mu_e)\beta\}} \qquad (21)
\end{aligned}
$$

$$\frac{\sigma_{\nu p}(B)}{V} = \frac{2G_F^2 C_A^2 eB}{(2\pi)^3}\int_0^\infty dp_{2Z} f_2(E_2)(E_\nu + \frac{p_{2Z}^2}{2m_p})\Bigg[(E_\nu + \frac{p_{2Z}^2}{2m_p}) \{2 - \frac{1}{3m_p}(E_\nu + \frac{p_{2Z}^2}{2m_p})\} - \frac{p_{nuZ}^2 + p_{2Z}^2}{m_p}\Bigg] \quad (22)$$

which can be approximated by

$$\frac{\sigma_{\nu p}(B)}{V} \approx \frac{G_F^2 C_A^2}{\pi} n_p E_\nu^2 \quad (23)$$

where

$$n_p = \frac{2eB}{(2\pi)^2}\int_0^\infty f_2(E_2) dp_{2Z} \quad (24)$$

$$\frac{\sigma_{\nu e}(B)}{V} \sim \frac{4G_F^2 eB}{(2\pi)^3}\mu_e T\Bigg[(C_V^2 + C_A^2)(1 + cos^2\theta) - 4C_V C_A cos\theta\Bigg] \frac{E_\nu}{1 + exp(-\mu_e/T)} \quad (25)$$

The neutrino-neutron scattering cross-section remains unchanged except through its dependence on neutron density which gets affected by the magnetic field.

For the noninteracting, hot n-p-e matter in beta-equilibrium we have the number densities given by

$$\bar{n}_e = n_{e^-} - n_{e^+} = \frac{2eB}{(2\pi)^2}\Bigg[\sum_\nu (2 - \delta_{\nu,o})\int_0^\infty dp_Z \frac{1}{1 + exp\{\beta(E_{e\nu} - \mu_e)\}} - (\mu_e \to -\mu_e)\Bigg] \quad (26)$$

$$n_p = \frac{2eB}{(2\pi)^2} \Bigg[\sum_\nu (2 - \delta_{\nu,o})\int_0^\infty dp_Z \frac{1}{1 + exp\{\beta(E_{p\nu} - \mu_p)\}}\Bigg] \quad (27)$$

$$n_n = \frac{1}{\pi^2}\int_0^\infty p^2 dp \frac{1}{1 + exp\{\beta(E_{n\nu} - \mu_n)\}} \quad (28)$$

The energy expressions in the presence of magnetic field are given in eqn(9). Charge neutrality requires $\bar{n}_e = n_p$ and β-equilibrium relates the chemical potentials viz. $\mu_n = \mu_p + \mu_e$

In Fig.1 we have plotted the proton fraction $Y_p = \frac{n_p}{n_n+n_p}$ as a function of density at two different temperatures 1 MeV and 10 MeV respectively for different values of the magnetic field. The neutron fraction is given by $Y_n = 1 - Y_p$. At T=1 MeV, there are very small number of positrons present and the effect of the magnetic field is to raise the proton fraction so much so that at $B = 10^4 MeV$, the matter becomes predominantly proton rich. This had the effect of not only modifying neutrino cross-sections quantitatively but to change the relative importance of various processes in the presence of magnetic field. In Fig.2 we have plotted the neutrino mean free path in meters as a function of baryon density for different values of the magnetic fields for matter at T=1 MeV. The neutrino energy is taken to be $E_1 = 3T$. We observe as expected that neutrino absorption cross section dominates the mean free path even in the presence of highest magnetic fields considered here as it does in the free field case. We also find that the absorption mean free path rises rapidly with density since at low temperatures due to Pauli-Blocking effect, the absorption process gets suppressed. At higher temperatures, there are lots of positrons present and electron degeneracy decreases and the effect is absent as can be seen from Figures 3 and 4 where we have plotted the mean free paths for matter at T=10 and T=30 MeV respectively. In fact at these temperatures, the free path decreases with increase in density. The absorption and neutrino electron scattering cross sections develop asymmetry when the matter is polarised as can be seen from eqns.(21) and(25). The

asymmetry in the absorption cross section for neutrinos propagating along and opposite to the direction of the magnetic field is roughly of the order of 20 %. The asymmetry is far more pronounced in neutrino electron scattering , so much so that scattering cross section for neutrinos in the direction of the magnetic field is ten times more compared to neutrinos progating parallel to the field. Thus at low temperatures, when the matter is highly polarised the neutrino sphere itself gets severely modified and distorted. This results in neutrinos along the magnetic field being emitted from deeper layers where the temperature is higher in comparison to neutrinos which are emitted anti parallel to the field direction. A change in cross sections (by an order of magnitude even at high temperatures) caused by a large magnetic field will change the density near the neutrino sphere considerably. We can see from Figures 2, 3 and 4 that increase in neutrino cross section is roughly proportional to B (albeit for polarised matter) thereby shifting the neutrino sphere to larger radii and hence to lower densities. This will have implications for the implementation of Kusenko-Segre mechanism to explain the observed pulsar velocities.

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References

[1] A.Vilenkin,*Astroph.J.* **451**(1995)700 A.V.Kuznestov and N.V.Mikhev, hep-ph/9612312 G.S.Bisnovatyi-Kogan, hep-ph/9707120

[2] V.G.Bezchastnov and P.Haensel, *Phys.Rev.***D54**(1996)3706

[3] C.J.Horowitz and J.Piekarewicz, hep-ph/9701214

[4] A.Kusenko and Gino Segre,*Phys.Rev.Lett.* **77**, 4872 (1996)

[5] H.Nunokawa,V.B.Semikoz,A.Yu.Smirnov and J.W.F.Valle, *Nucl.Phys.* **B501** , 17(1997)

[6] N.N.Chugai,*Sov.Astron.Lett.***10**, 87 (1984)

[7] E.Mueller and W.Hillebrandt,*Astrn.and Astroph.* **80**, 147 (1979) C.Thompson and R.C.Duncan,*Astrophy.J.***392**, 19 (1992) G.S.Bisnovatyi and S.G.Moiseenko,*Sov.Astron.***36**, 285 (1992) M.Bocquet et.al.,*Astron. and Astrophy.***301**, 757 (1995)

[8] Esteban Roulet, hep-ph/9711206

[9] J.J.Matese and R.F.Connell,*Phys.Rev.***180**, 1289 (1969) L.Fassio-Canuto,*Phys.Rev.***187**, 2141 (1969) D.Lai and S.L.Shapiro,*Astroph.J.***383**, 745 (1991)

[10] C.Itzykson and J-B.Zuber,*Quantum Field Theory* (McGraw Hill,New York,1985) p.67

[11] See for example D.L.Tubbs and D.N.Schramm,*Astrophys.J.***201**, 467 (1975) A.Burrows and J.M.Lattimer,*Astrophys.J.***307**, 178 (1986)

Figure Captions

Fig.1 Proton fraction Y_p as a function of baryon density in gm/cc for matter at $T = 1$ MeV and $T = 10$ MeV respectively. The solid, long dashed, dashed and dotted curves are for $B = 0, 10^2, 10^3$ and $10^4 MeV^2$ respectively.

Fig.2 Neutrino mean free path in meters as a function of baryon density for matter at T=1 MeV and $B = 0, 10^2$ and $10^4 MeV^2$. The neutrino energy is taken to be 3T. The solid curve is for the absorption process and the long dashed, dashed and dotted curves are for neutrino scattering on neutrons, protons and electrons respectively.

Fig.3 Neutrino mean free path in meters as a function of baryon density for matter at T=10 MeV and $B = 0$ and $10^4 MeV^2$. The neutrino energy is taken to be 3T. The solid, long dashed, dashed and dotted curves are as in Figure2.

Fig.4 Neutrino mean free path in meters as a function of baryon density for matter at T=30 MeV and $B = 0$ and $10^4 MeV^2$. The neutrino energy is taken to be 3T. The solid curve, long dashed, dashed and dotted curves are as in Figure2.

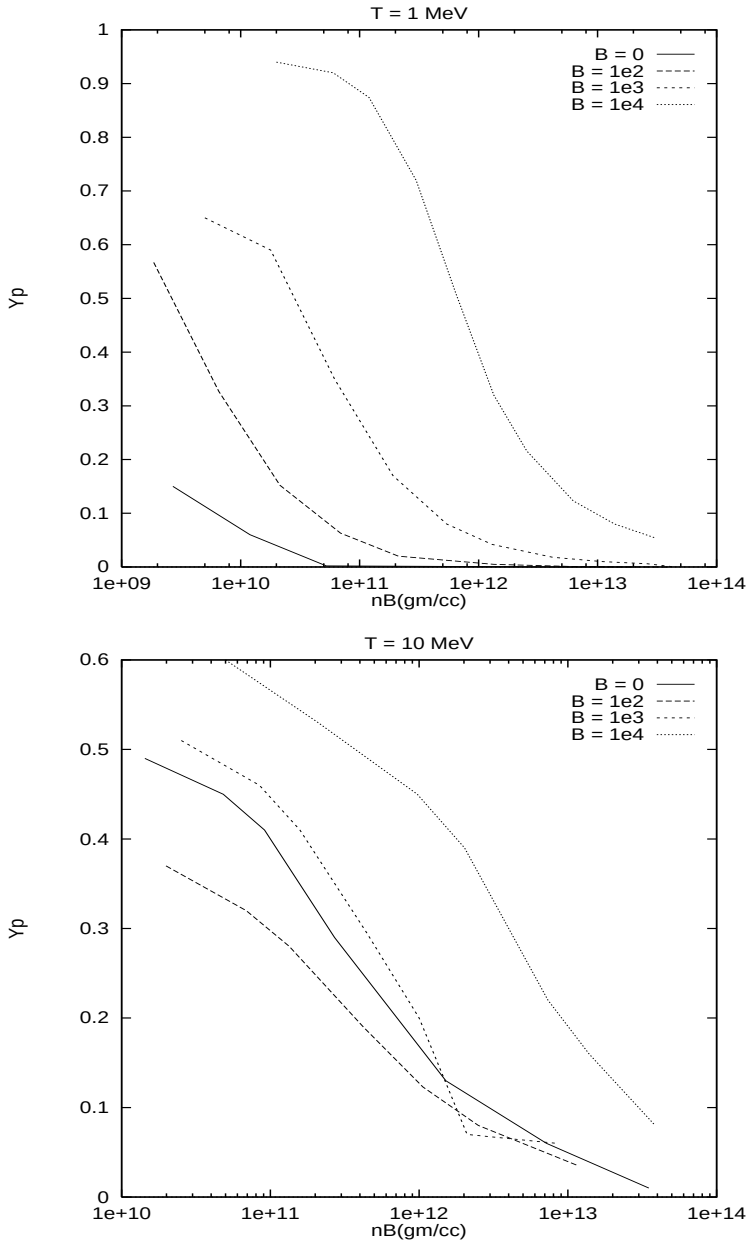
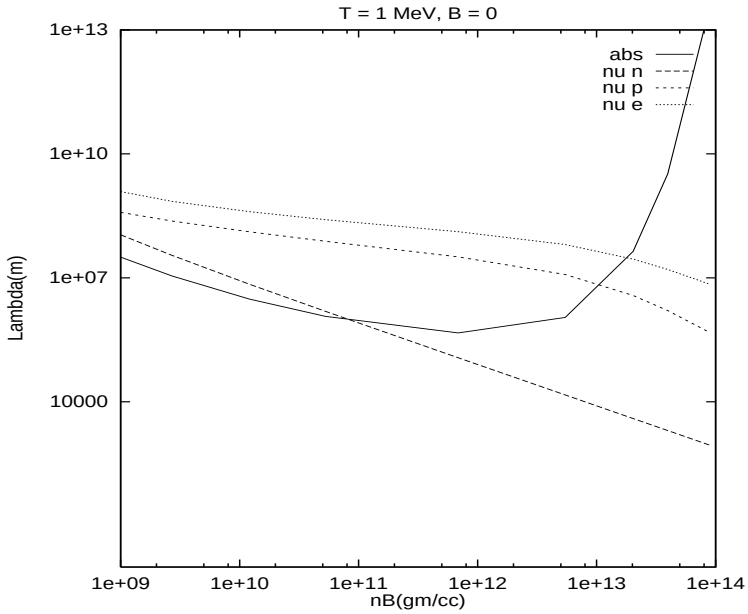

T = 1 MeV
B = 0
B = 1e2
B = 1e3
B = 1e4
Yp
nB(gm/cc)
1e+09
1e+10
1e+11
1e+12
1e+13
1e+14
T = 10 MeV
B = 0
B = 1e2
B = 1e3
B = 1e4
Yp
nB(gm/cc)
1e+10
1e+11
1e+12
1e+13
1e+14

Figure 1: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1e+13
1e+10
1e+07
10000
Lambda(m)
abs
nu n
nu p
nu e
1e+09
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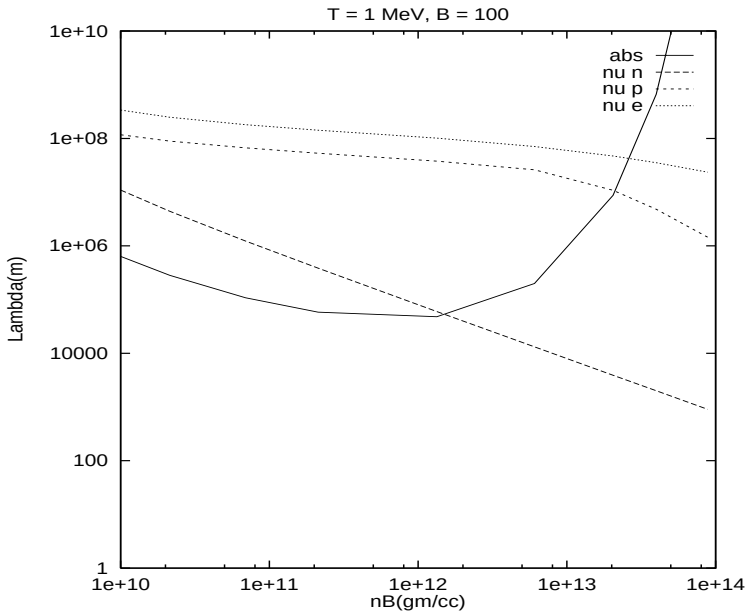
1e+12
1e+13
1e+14
nB(gm/cc)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1e+10
1e+08
1e+06
10000
100
1
Lambda(m)
abs
nu n
nu p
nu e
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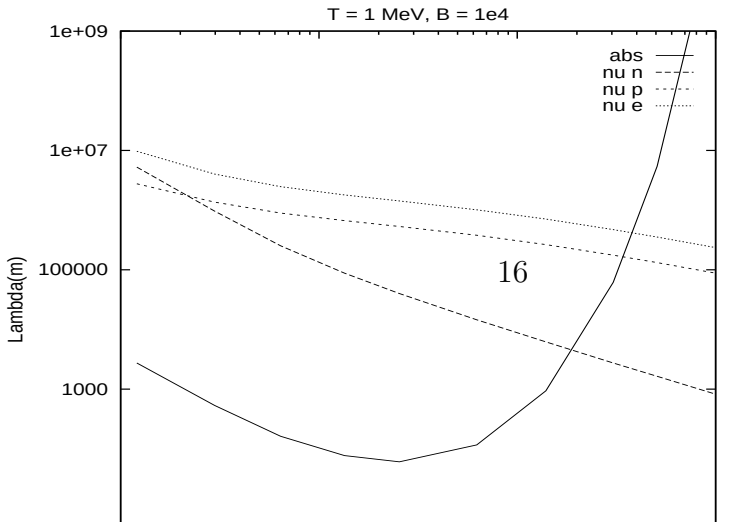
1e+12
1e+13
1e+14
nB(gm/cc)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1e+07
100000
1000
Lambda(m)
abs
nu n
nu p
nu e

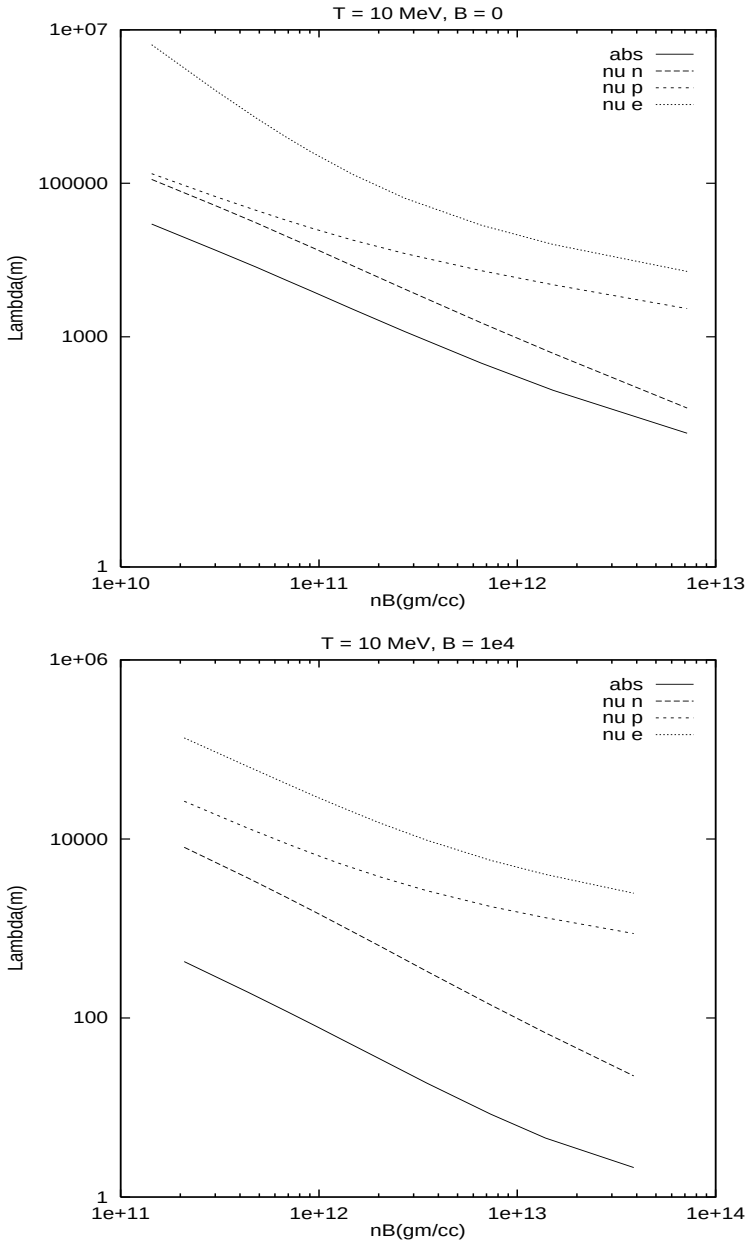
T = 10 MeV, B = 0
1e+07
100000
1000
1
Lambda(m)
1e+10
1e+11
1e+12
1e+13
nB(gm/cc)
abs
nu n
nu p
nu e
T = 10 MeV, B = 1e4
1e+06
10000
100
1
Lambda(m)
1e+11
1e+12
1e+13
1e+14
nB(gm/cc)
abs
nu n
nu p
nu e

Figure 3:

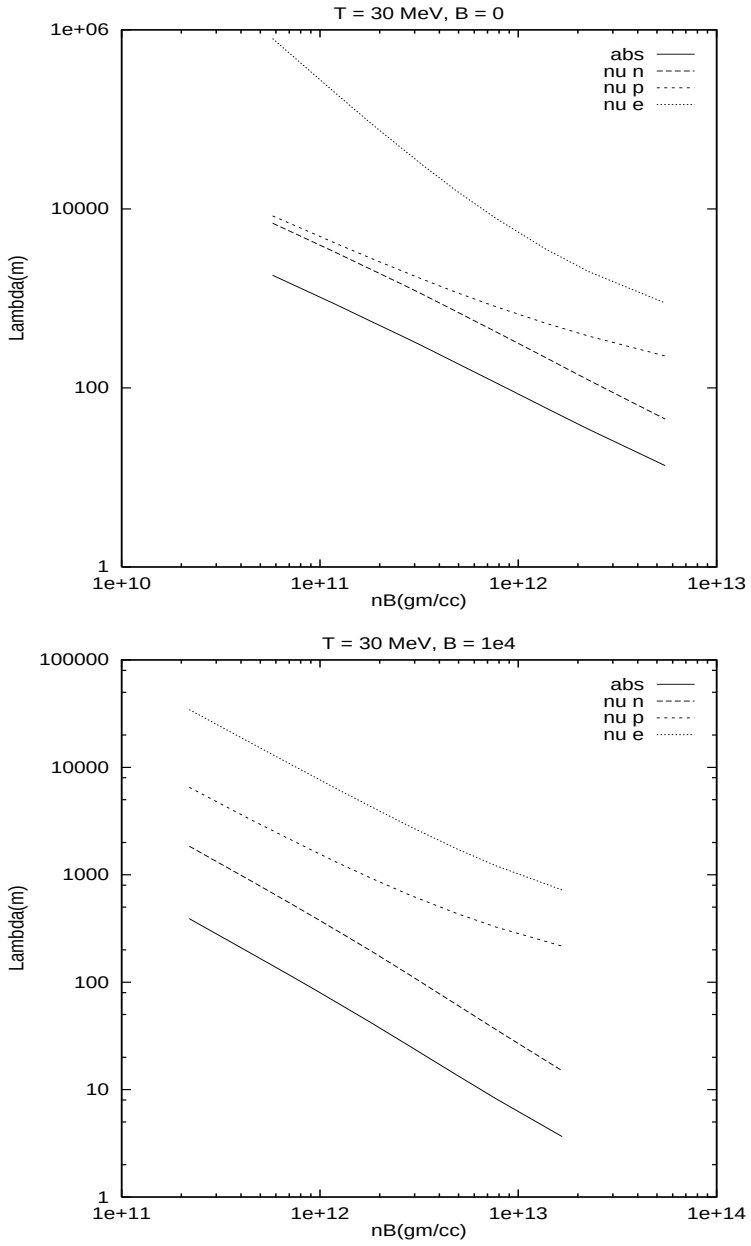
T = 30 MeV, B = 0
1e+06
10000
100
1
Lambda(m)
1e+10
1e+11
1e+12
1e+13
nB(gm/cc)
abs
nu n
nu p
nu e
T = 30 MeV, B = 1e4
100000
10000
1000
100
10
1
Lambda(m)
1e+11
1e+12
1e+13
1e+14
nB(gm/cc)
abs
nu n
nu p
nu e

Figure 4: